Operations with Rational Expressions

~ Recognizing the lowest common denominator of algebraic expressions.

Let's start with something easy:
Find the lowest common denominator of

a) 2 and 3

b) 6 and 5

Isn't the easiest and most sure fire method of finding a common denominator simply multiplying one by another????

So try to find the common denominator of the following....

a) x and 4x $4x^2$ $4x$

b) 24r and 8r $32r$ $8r$

c) xy and yz xy^2z xyz

To multiply rational expressions we always multiply the numerators with the numerators and the denominators with the denominators.

To divide rational expressions we always multiply the first term by the reciprocal of the second term.

So how to we add or subtract rational expressions????

All we do is change each expression to an equivalent one with a common denominator. Once the denominators are the same, the numerators can be added or subtracted.

For example:

$$\frac{5}{5}\cdot\frac{3}{x}+\frac{2}{5x}=\frac{15}{5x}+\frac{2}{5x}=\frac{17}{5x}$$

Practice:

1) Express $\frac{2}{x+1} - \frac{1}{x-1}$ as a single fraction. Assume that x does not equal 1 or -1.

$$\frac{(x-1)}{(x-1)}\frac{2}{x+1} - \frac{1}{(x-1)}\left(\frac{x+1}{x+1}\right)$$

$$= \frac{2(x-1) - 1(x+1)}{(x-1)(x+1)}$$

$$= \frac{2x-2 \overset{+}{-} (x \overset{-}{+} 1)}{(x-1)(x+1)} = \frac{x-3}{x^2+x-x-1}$$

$$= \frac{x-3}{x^2-1}$$

2) Divide $\frac{2}{x+1} \div \frac{1}{x-1}$ Assume that x does not equal 1 or -1.

$$\frac{2}{X+1} \cdot \frac{X-1}{1} = \frac{2(X-1)}{X+1}$$

$$= \frac{2X-2}{X+1}$$

3) Multiply $\frac{b^3c^7}{a^3d^4} \cdot \frac{a^2d^3}{abc^4}$ Assume none of the variables equals 0.

$$\frac{b^3c^7a^2d^3}{a^4bc^4d^4} = \boxed{\frac{b^2c^3}{a^2d}}$$

4) Divide $\dfrac{4x^3y^2}{-9r^4c^2} \div \dfrac{x^2}{-3rc}$ Assume r and c do not equal 0.

$$\frac{4x^3y^2}{-9r^4c^2} \cdot \frac{-3rc}{x^2} = \frac{-12x^3y^2rc}{-9r^4c^2x^2}$$

$$= \frac{4xy^2}{3r^3c}$$

5) Simplify: $\dfrac{x}{4+x} - \left(\dfrac{-4}{4+x}\right)$

$$\frac{X + (+4)}{4+X} = \frac{X+4}{4+X} = 1$$

⑤ $\frac{a}{b}\left(\frac{a}{a}\right) - \frac{1}{a}\left(\frac{b}{b}\right)$

$$\frac{a^2 - b}{ab}$$

6) Express as a single fraction: $\frac{n}{n}\frac{2p}{np^2} + \frac{3n}{n^2p}\left(\frac{p}{p}\right)$

$$= \frac{2pn + 3pn}{n^2p^2}$$

$$= \frac{5pn}{n^2p^2} = \frac{5}{pn}$$