

Operations with Rational Expressions

~ Recognizing the lowest common denominator of algebraic expressions.

Let's start with something easy:

Find the lowest common denominator of

a) 2 and 3

b) 6 and 5

Isn't the easiest and most sure fire method of finding a common denominator simply multiplying one by another????

So try to find the common denominator of the following....

a) x and $4x$ $4x^2$ $4x$

b) $24r$ and $8r$ $32r$ $8r$

c) $2xy$ and yz xy^2z xyz

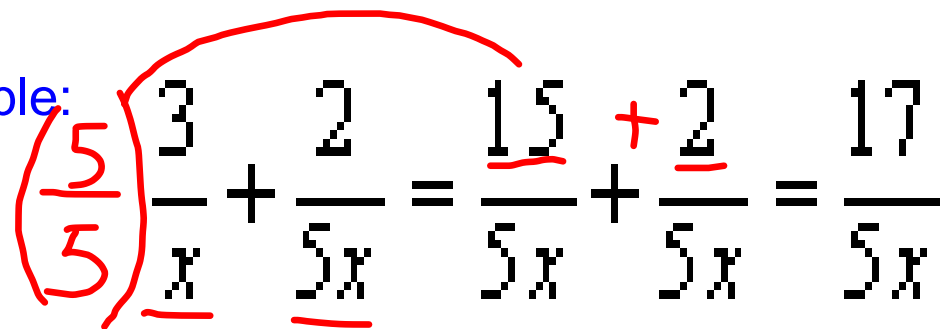
To multiply rational expressions we always multiply the numerators with the numerators and the denominators with the denominators.

To divide rational expressions we always multiply the first term by the reciprocal of the second term.

So how to we add or subtract rational expressions???

All we do is change each expression to an equivalent one with a common denominator. Once the denominators are the same, the numerators can be added or subtracted.

For example:



The equation shows the addition of two rational expressions. A red bracket on the left groups the two fractions. A red arc connects the denominator of the first fraction to the denominator of the second fraction, indicating the common denominator. The numerators are also underlined in red.

$$\left(\frac{3}{x} + \frac{2}{5x} \right) = \frac{15}{5x} + \frac{2}{5x} = \frac{17}{5x}$$

Practice:

1) Express $\frac{2}{x+1} - \frac{1}{x-1}$ as a single fraction. Assume that x does not equal 1 or -1.

$$\begin{aligned} & \frac{(x-1) \frac{2}{x+1}}{(x-1)} - \frac{1}{(x-1)} \left(\frac{x+1}{x+1} \right) \\ &= \frac{2(x-1) - 1(x+1)}{(x-1)(x+1)} \\ &= \frac{2x - 2 - (x + 1)}{(x-1)(x+1)} = \frac{x - 3}{x^2 + x - x - 1} \\ &= \boxed{\frac{x - 3}{x^2 - 1}} \end{aligned}$$

2) Divide $\frac{2}{x+1} \div \frac{1}{x-1}$ Assume that x does not equal 1 or -1.

$$\frac{2}{x+1} \cdot \frac{x-1}{1} = \frac{2(x-1)}{x+1}$$

$$= \frac{2x-2}{x+1}$$

3) Multiply $\frac{b^3 c^7}{a^3 d^4} \cdot \frac{a^2 d^3}{abc^4}$ Assume none of the variables equals 0.

$$\frac{b^3 c^7 a^2 d^3}{a^4 b c^4 d^4} = \boxed{\frac{b^2 c^3}{a^2 d}}$$

4) Divide $\frac{4x^3y^2}{-9r^4c^2} \div \frac{x^2}{-3rc}$ Assume r and c do not equal 0.

$$\frac{4x^3y^2}{-9r^4c^2} \cdot \frac{-3rc}{x^2} = \frac{-12x^3y^2rc}{-9r^4c^2x^2}$$

$$= \frac{4xy^2}{3r^3c}$$

5) Simplify: $\frac{x}{4+x} - \left(\frac{-4}{4+x} \right)$

$$\frac{X + (+4)}{4+X} = \frac{X+4}{4+X} = 1$$

$$\textcircled{5} \quad \frac{a}{b} \left(\frac{a}{a} \right) - \frac{1}{a} \left(\frac{b}{b} \right)$$

$$\frac{a^2 - b}{ab}$$

6) Express as a single fraction:

$$\frac{n}{h} \frac{2p}{np^2} + \frac{3n}{n^2 p} \left(\frac{p}{p} \right)$$

$$= \frac{2pn + 3pn}{h^2 p^2}$$

$$= \frac{5pn}{h^2 p^2} = \frac{5}{pn}$$